ETE 2200

Enhanced Guided Notes: Set 14

Resonance (Series and Parallel Circuit)

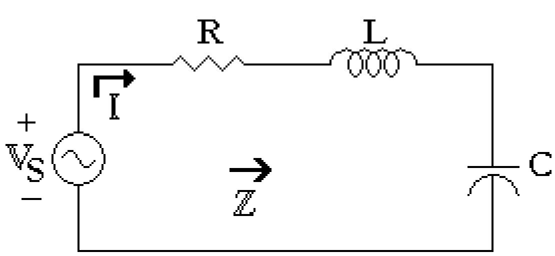
Topics:

A. Series Resonance Circuit
B. Parallel Resonant Circuit

What is a resonant (tuned) circuit?

How to build a resonant (tuned) circuit?

A. Series Resonance Circuit



Condition:

$$\mathbf{X_L = X_C}$$

It means

$$\mathbb{Z}_{T_S} = \mathbf{R}$$

How to find the resonant frequency

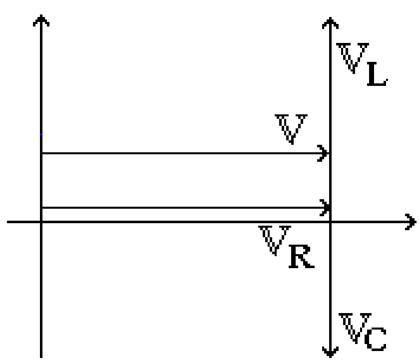
$X_L = X_C$

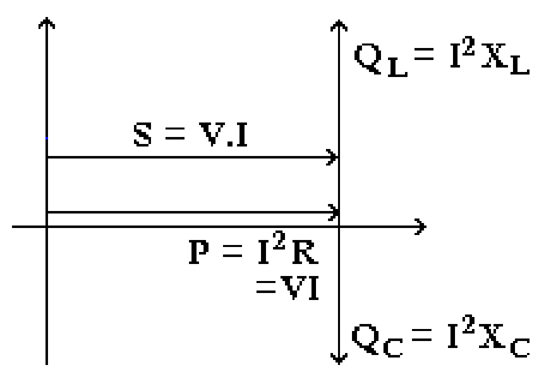
$\omega L = 1/$ $\quad\quad\quad\quad$ ~> $\quad \omega^2 = 1/$

$\therefore \quad \omega_S = $ _______

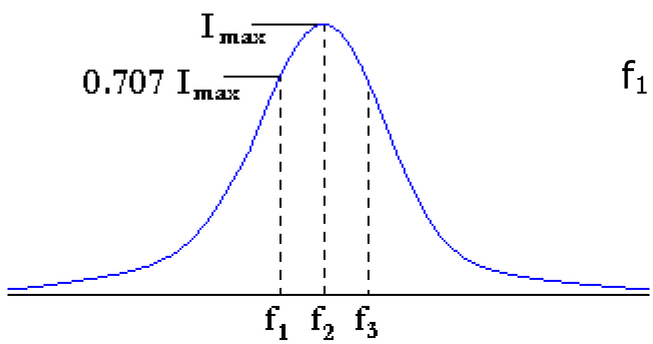
What can we say about (at the resonant freq.)

- $\mathbb{I}$ =
- $\mathbb{Z}_T$ =
- $\mathbb{V}_L$ =
- $\mathbb{V}_C$ =
- Phasor diagram





$\therefore F_P = \quad \cos\theta \quad = \quad P/S \quad = \quad 1$



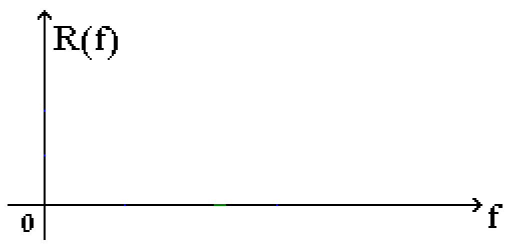
f_1 or f_2 $\equiv$ band freq

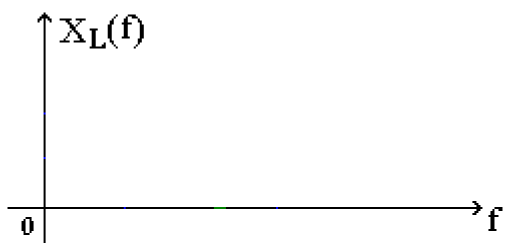
$\equiv$

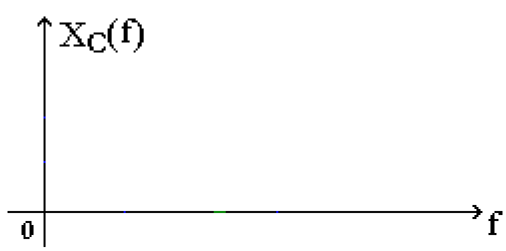
$\equiv$

$\equiv$

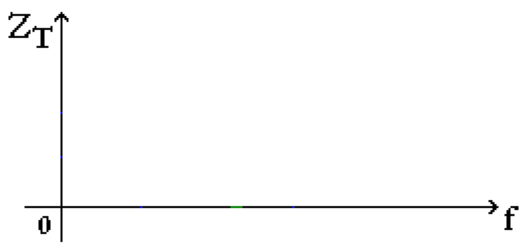
Now, when we evaluate Z_T versus frequency





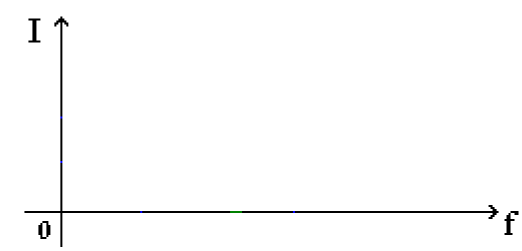


$F_S \equiv$



Is the Z_T curve symmetrical?

Now, if we plot I vs. f



$$I = \frac{V}{Z_T}$$

$$P_{max} = I^2_{max}R$$

$$P_{HPF} = I^2R \qquad = (0.707\ I_{max})^2R$$

$$= 0.5\ I^2_{max}R \qquad = \tfrac{1}{2}\ P_{max}$$

$\therefore \quad P_{HPF} = \frac{1}{2} P_{max}$

The Quality factor (Q)

↓

indicates how much energy is placed in storage compared to that dissipated

$$Q_S = \frac{\text{Reactive power}}{\text{Average power}}$$

$$Q_S = \frac{I^2 X_L}{I^2 R} = \frac{X_L}{R} = \frac{\omega_S L}{R}$$

Bandwidth (BW)

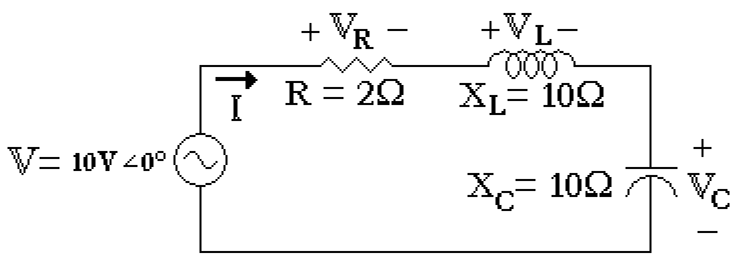
$BW = f2 - f1 = R / 2\pi L$

$BW = f_S / Q_S$

____________ : f_S

$BW / f_S = 1 / Q_S$

$$\frac{f_2 - f_1}{f_S} = \frac{1}{Q_S}$$



a. Find $\mathbb{I}$, $\mathbb{V}_R$, $\mathbb{V}_L$, and $\mathbb{V}_C$ at resonance

First let's ask ourselves:

- ★ Is it a series resonant circuit?

- ★ Why?

$\therefore\ \mathbb{Z}_{T_S}\ =$

And $\mathbb{I} = \mathbb{V}/\mathbb{Z}_{T_S}\ =$

$\mathbb{V}_R\ =$

$\mathbb{V}_L\ =$

$\mathbb{V}_C\ =$

b. Find the Q_S of the circuit

$$Q_S = \frac{X_L}{R} = \frac{X_C}{R} = \qquad =$$

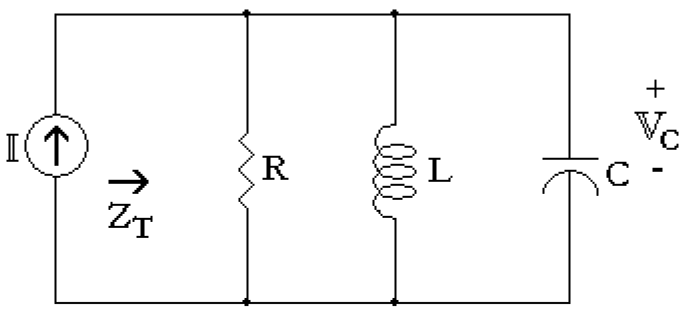
c. If the resonant frequency is 5000 Hz, find the bandwidth.

BW = = =

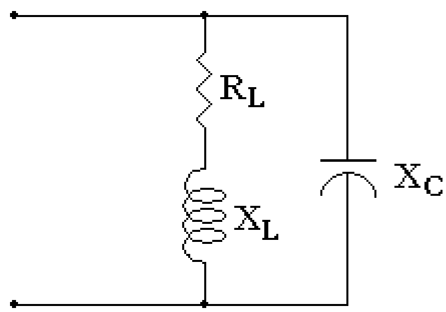
d. Find the power dissipated in the circuit at half power frequencies.

$$P_{HPF} = \frac{1}{2} P_{max}$$

B. Parallel Resonant Circuit



← Ideal



← Practical

$$\therefore \quad Z_{R\text{-}L} = R_L + j X_L$$

$$Y_{R\text{-}L} = \frac{1}{Z_{RL}} = \frac{1}{R_L + jX_L} = \frac{1}{R_L + jX_L}\,\frac{R_L - jX_L}{R_L - jX_L}$$

$$=$$

Condition: $X_C = X_L$

Earlier, we learned that $X_L = R_\ell^2 + X_L^2 / X_L$

$$\therefore \quad \frac{R_\ell^2 + X_L^2}{X_L} = X_C$$

$$R_\ell^2 + X_L^2 = X_C X_L$$

$$= (\omega L)\left(\frac{1}{\omega C}\right) = \frac{L}{C}$$

$$\therefore \quad R_\ell^2 + X_L^2 = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R_\ell^2$$

$$2\pi f L = \sqrt{\frac{L}{C} - R_\ell^2}$$

$$f = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_\ell^2} \qquad = \frac{1}{2\pi L}\sqrt{\left(\frac{L - R_\ell^2 C}{C}\right)\left(\frac{C/L}{C/L}\right)}$$

$$= \frac{1}{2\pi L}\sqrt{\frac{(LC/L) - (R_\ell^2 C^2/L)}{C^2/L}}$$

$$= \frac{1}{2\pi L}\sqrt{\frac{1 - R_\ell^2 C/L}{C/L}}$$

$$= \frac{1}{2\pi L\sqrt{C/L}}\sqrt{1 - \frac{R_\ell^2 C}{L}}$$

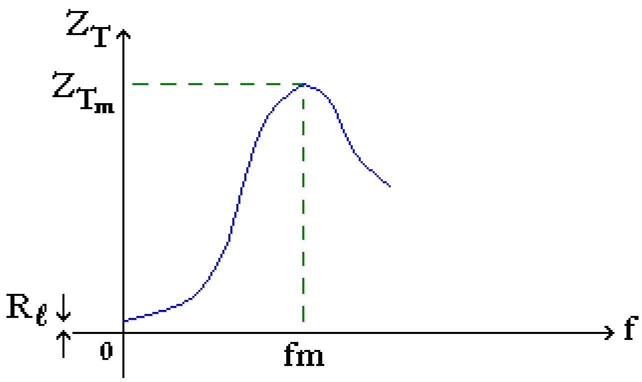
$$= \underbrace{\frac{1}{2\pi\sqrt{LC}}}_{f_S}\sqrt{1 - \frac{R_\ell^2 C}{L}}$$

$$\therefore \quad f_p = f_S \sqrt{\qquad\qquad}$$

What happens at $f = f_p$?

Maximum impedance at fm

$$fm = fs \sqrt{1 - \frac{1}{4}\left(\frac{R_\ell^{\,2} C}{L}\right)}$$



What happens if $R_\ell = 0$?

$$f_p =$$

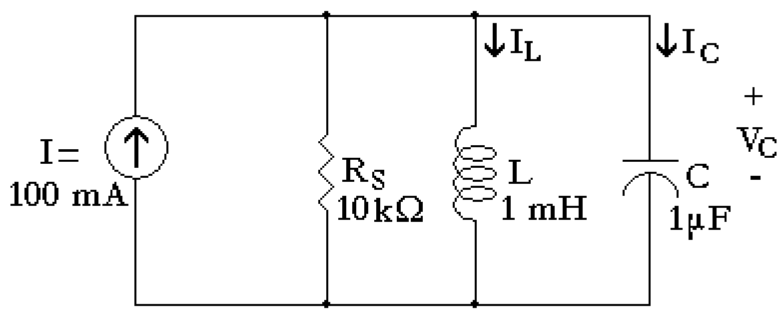
$$Q_p = \frac{R}{X_{L_p}} = \frac{R_S // R_p}{X_{L_p}} =$$

$$BW = f_2 - f_1 = \frac{f_p}{Q_p}$$

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\ \right]$$

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\ \right]$$

✵



a. Find f_p

$f_p = f_S =$

b. Find the total impedance at resonance

$\mathbb{Z}_L // \mathbb{Z}_C =$ ______________________________

c. Quality factor $Q_p = \dfrac{R_S}{X_{L_P}} =$

$$BW = \frac{f_P}{Q_P}$$

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

$=$

$=$

d. V_C at Resonance

$$V_C = I_{Z_{Tp}}$$

$=$

e. $I_L = V_L / X_L$

$I_C =$

Are I_L and I_C the same at resonance frequency?