

Enhanced Guided Notes: Set 14

Resonance (Series and Parallel Circuit)

Topics:

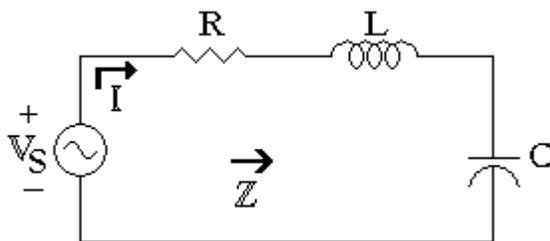
- A. Series Resonance Circuit
- B. Parallel Resonant Circuit

What is a resonant (tuned) circuit?



How to build a resonant (tuned) circuit?

A. Series Resonance Circuit



Condition:

$$X_L = X_C$$

It means

$$Z_{T_S} = R$$

How to find the resonant frequency

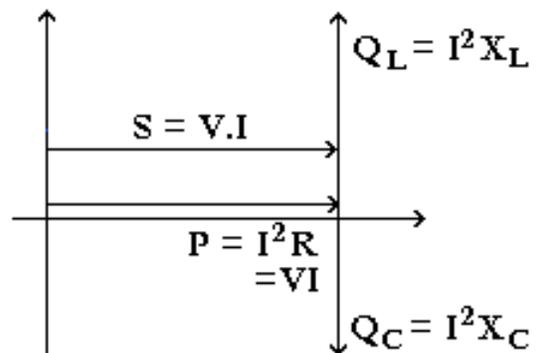
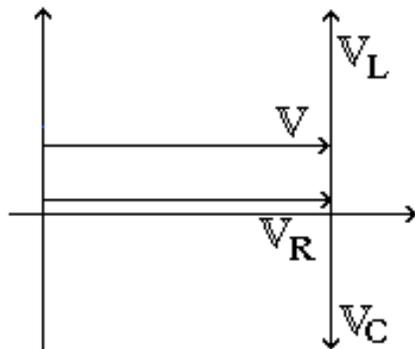
$$X_L = X_C$$

$$\omega L = 1/ \quad \sim > \quad \omega^2 = 1/$$

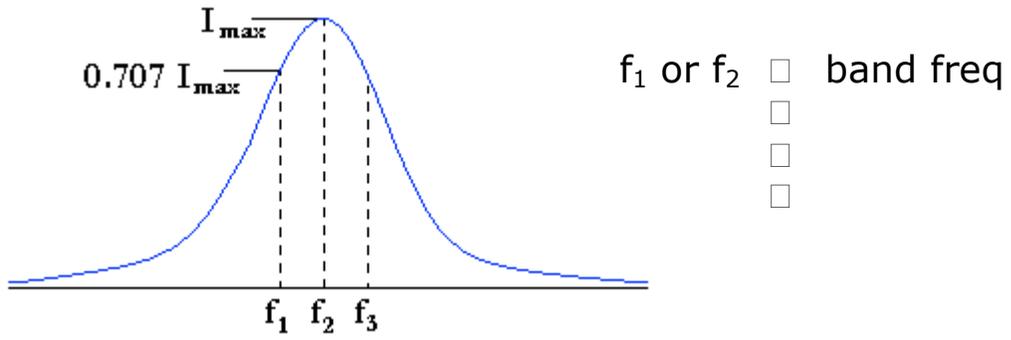
$$\therefore \omega_s = \underline{\hspace{2cm}}$$

What can we say about (at the resonant freq.)

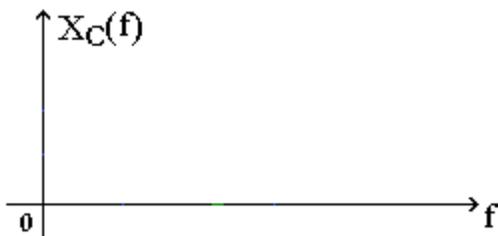
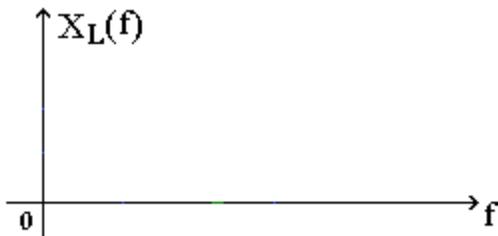
- $I =$
- $Z_T =$
- $V_L =$
- $V_C =$
- Phasor diagram

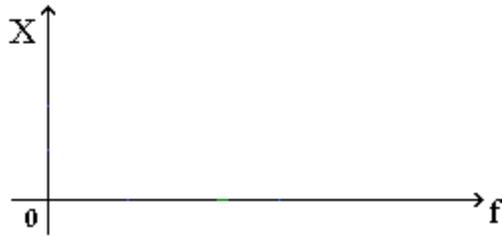


$$\therefore F_p = \cos \square = P/S = 1$$



Now, when we evaluate Z_T versus frequency





F_s □



Is the Z_T curve symmetrical?

Now, if we plot I vs. f



$$I = \frac{V}{Z_T}$$

$$P_{\max} = I_{\max}^2 R$$

$$\begin{aligned} P_{\text{HPF}} &= I^2 R &= (0.707 I_{\max})^2 R \\ &= 0.5 I_{\max}^2 R &= \frac{1}{2} P_{\max} \end{aligned}$$

$$\therefore P_{\text{HPF}} = \frac{1}{2} P_{\text{max}}$$

The Quality factor (Q)

↓

indicates how much energy is placed in storage compared to that dissipated

$$Q_s = \frac{\text{Reactive power}}{\text{Average power}}$$

$$Q_s = \frac{I^2 X_L}{I^2 R} = \frac{X_L}{R} = \frac{\omega_s L}{R}$$

Bandwidth (BW)

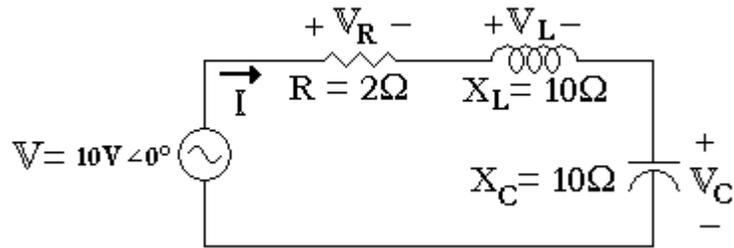
$$BW = f_2 - f_1 = \frac{R}{2\pi L}$$

$$BW = f_s / Q_s$$

$$\text{_____} : f_s$$

$$BW / f_s = 1 / Q_s$$

$$\frac{f_2 - f_1}{f_s} = \frac{1}{Q_s}$$



a. Find I , V_R , V_L , and V_C at resonance

First let's ask ourselves:

★ Is it a series resonant circuit?

★ Why?

$$\therefore Z_{T_S} =$$

$$\text{And } I = V/Z_{T_S} =$$

$$V_R =$$

$$V_L =$$

$$V_C =$$

b. Find the Q_S of the circuit

$$Q_S = \frac{X_L}{R} = \frac{X_C}{R} = \quad =$$

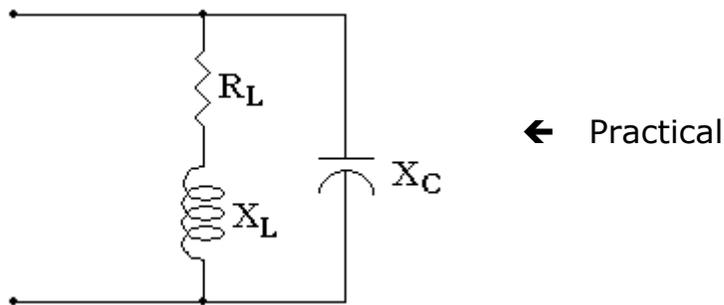
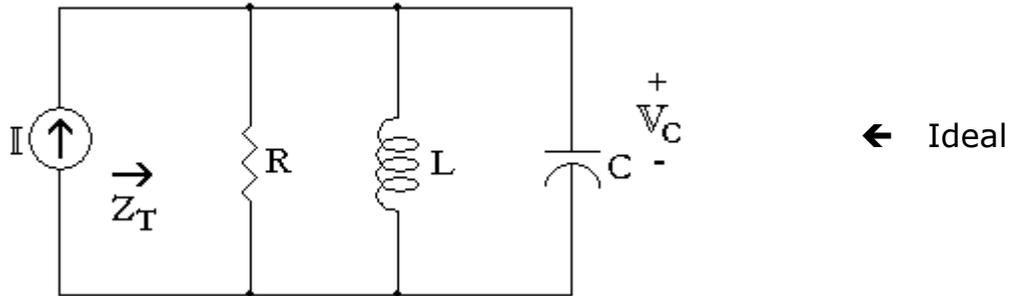
- c.** If the resonant frequency is 5000 Hz, find the bandwidth.

$$BW = \quad = \quad =$$

- d.** Find the power dissipated in the circuit at half power frequencies.

$$P_{\text{HPF}} = \frac{1}{2} P_{\text{max}}$$

B. Parallel Resonant Circuit



$$\therefore Z_{R-L} = R_L + j X_L$$

$$Y_{R-L} = \frac{1}{Z_{R-L}} = \frac{1}{R_L + j X_L} = \frac{1}{R_L + j X_L} \frac{R_L - j X_L}{R_L - j X_L}$$

=

Condition: $X_C = X_L$

Earlier, we learned that $X_L = R\ell' + X_L^2 / X_L$

$$\therefore \frac{R\ell'^2 + X_L^2}{X_L} = X_C$$

$$\begin{aligned} R\ell'^2 + X_L^2 &= X_C X_L \\ &= (\omega L) \left(\frac{1}{\omega C} \right) = \frac{L}{C} \end{aligned}$$

$$\therefore R\ell'^2 + X_L^2 = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R\ell'^2$$

$$2\pi fL = \sqrt{\frac{L}{C} - R\ell'^2}$$

$$f = \frac{1}{2\pi L} \sqrt{\frac{L}{C} - R\ell'^2} = \frac{1}{2\pi L} \sqrt{\left(\frac{L - R\ell'^2 C}{C}\right) \left(\frac{C/L}{C/L}\right)}$$

$$= \frac{1}{2\pi L} \sqrt{\frac{(LC/L) - (R\ell'^2 C^2/L)}{C^2/L}}$$

$$= \frac{1}{2\pi L} \sqrt{\frac{1 - R\ell'^2 C/L}{C/L}}$$

$$= \frac{1}{2\pi L \sqrt{C/L}} \sqrt{1 - \frac{R\ell'^2 C}{L}}$$

$$= \frac{1}{2\pi \sqrt{LC}} \sqrt{1 - \frac{R\ell'^2 C}{L}}$$

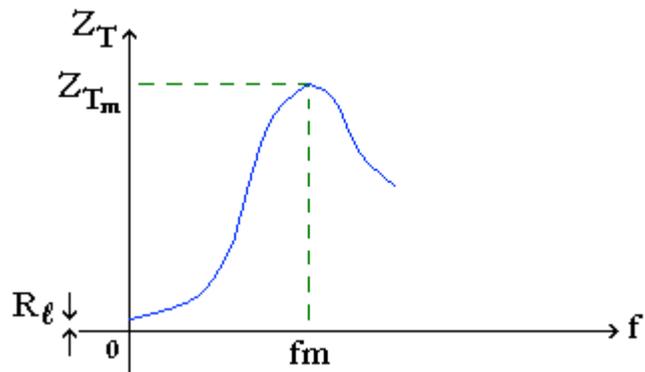
f_s

$$\therefore f_p = f_s \sqrt{\quad}$$

What happens at $f = f_p$?

Maximum impedance at f_m

$$f_m = f_s \sqrt{1 - \frac{1}{4} \left(\frac{R_\ell^2 C}{L} \right)}$$



What happens if $R_\ell = 0$?

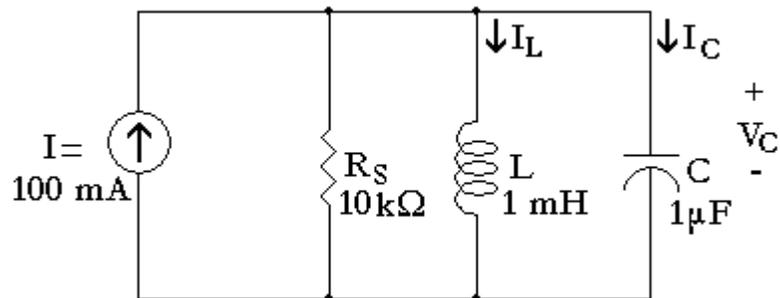
$$f_p =$$

$$Q_p = \frac{R}{X_{Lp}} = \frac{R_S // R_P}{X_{Lp}} =$$

$$BW = f_2 - f_1 = \frac{f_p}{Q_p}$$

$$f_1 = \frac{1}{4\pi C} \left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

$$f_2 = \frac{1}{4\pi C} \left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$



- a. Find f_p

$$f_p = f_s =$$

- b. Find the total impedance at resonance

$$Z_L // Z_C = \underline{\hspace{10cm}}$$

- c. Quality factor $Q_p = \frac{R_S}{X_{Lp}} =$

$$BW = \frac{f_p}{Q_p}$$

$$f_1 = \frac{1}{4\pi C} \left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

=

=

d. V_C at Resonance

$$V_C = I_{Z_{Tp}}$$

=

e. $I_L = V_L / X_L$

$$I_C =$$

Are I_L and I_C the same at resonance frequency?