ETE 2200

Enhanced Guided Notes: Set 10

Inductance

Topics:

- **A.** **Magnetic Field**
- **B.** **Inductance**
- **C.** **Induced Voltage**
- **D.** **R-L Transients: The Storage Phase**
- **E.** **Initial Conditions**
- **F.** **R-L Transients: The Release Phase**
- **G.** **Instantaneous Values**
- **H.** **Average Induced Voltage**
- **I.** **Inductors in Series and Parallel**
- **J.** **Steady State Conditions**
- **K.** **Energy Stored by and Inductor**

A. Magnetic Field

- Electromagnetism is a magnetic effect induced by the _______

- Magnetic flux lines have ______________ loops that travel through the material

    ★ How is this different from electrical flux lines?

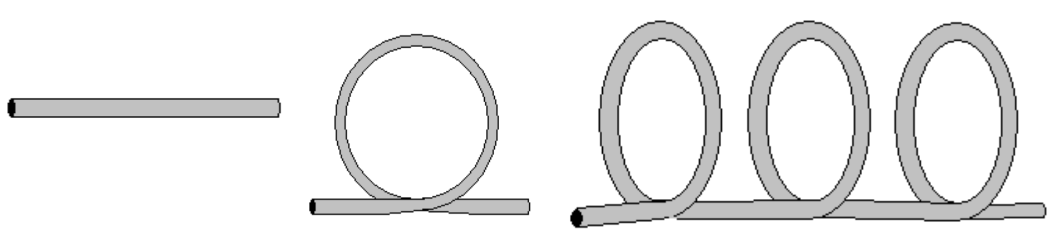
- Magnetic field is stronger near the ____________

- Magnetic flux lines pass easier through magnetic materials than air.
    This principle is used in ______________ sensitive material components.

- Electrical flux will exist around any conductor passing _______

    - flux direction is defined by right hand rule

★ Draw and label with arrowheads the flux lines associated with the following objects:



- Flux lines leaving coil from left and entering right simulate a

  _____________ and _____________ pole.

- This process is _______ than a magnet but can be increased

  by placing a core in the windings making an _____________

- Magnetic flux is measured in ____________ (Wb) designated with Φ (Greek *Phi*)

Flux Density

(SI system)	B = Wb/m^2 = Teslas (T)	"1 tesla = 1T = 1 Wb/m^2"
	Φ = webers (Wb)	1 weber of magnetic flux passes
$B = \Phi / A$	A = m^2	through an area of 1m2 yields a flux density of 1 Tesla

Magnetomotive Force

$\mathcal{F} = NI$ I = current (A)
N = number of coils

★ A greater number of coils will do what to the magnetomotive force?

- Permeability (μ) is a ___________ of the ease of establishment of magnetic flux lines in the material.

$\mu_0 = 4\pi \times 10^{-7}$ Wb/A·m for air

diamagnetic =

ferromagnetic =

paramagnetic =

Relative Permeability

$$\mu_r = \frac{\mu}{\mu_0}$$

Update in book
It's wrong
pg. 465

Ferromagnetic materials $\mu_r \geq 100$
Non-magnetic materials $\mu_r = 1$

Can't provide table of values because μ_r is a function of operating conditions.

B. Inductance

- Measured in ___________ (H) but most inductors will be mH or μH.

- __________ Henry is the inductance level to establish 1 Volt potential difference across the coil due to a change in current

  of _______ A/S through the coil.

- Dependant upon area within coil, length of the unit, and the

  _______________ of the core material, and the number of turns in the coil.

★ Why does the number of turns in the coil affect inductance?

$$L = \frac{\mu N^2 A}{\ell}$$

L = henries (H)
ℓ = length in m
A = core area in m^2
N = number of turns
μ = permeability (Wb/A·m)

★ Increasing the number of turns a large degree while decreasing the wire diameter to maintain a small inductor size has a limiting factor. What limits this and why?

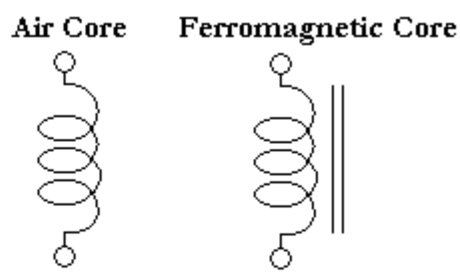
Replacing μ with $\mu = \mu_r \mu_0$ yields

$$L = \frac{\mu_r \mu_0 N^2 A}{\ell} \qquad \text{or} \qquad L = 4\pi \times 10^{-7} \frac{\mu_r N^2 A}{\ell}$$

$$\downarrow$$

$$L = \mu_r \underbrace{\left(\frac{\mu_0 N^2 A}{\ell}\right)}_{L_0} \rightarrow L = \mu_r L_0$$

★ What does this last equation tell us?



Types of Inductors

Fixed

Size is determined primarily by the type of construction, the core used, or the current rating

Phenolic inductor is pretty small for inductance level

DC resistance of inductor increases with a decrease in the thickness of the wire.

Variable

Inductance changed by moving core in and out of the windings.

Practical Equivalent Inductors

Inductors have losses due to resistance in wires and stray capacitances from parallel wires separated by air.

Inductor Labeling

Larger units will have it printed on the case.

Smaller units

Fundamental unit is μH

223k - 3rd number is multiplier

K denotes tolerance of ________%
J denotes tolerance of ±5%
M denotes tolerance of ±20%

Color code like resistors but result is always in μH

Inductance meter is best choice to measure inductor.

__________________ can check whether a ________ has developed between windings or whether an open circuit has developed.

Examples:

A coil with an air core measures 1″ in length with 200 turns of ½″ diameter. Find the inductance with the air core and compare it to the inductance with metallic core inserted. The metallic core has a relative permeability.

B. Induced Voltage

Equations Affecting Transient Response

- Faradays law of electromagnetic induction

  - Moving a ___________ through a __________ _________

    so it cuts magnetic lines of flux induces a ____________

    across the conductor. Faster movement causes _______ voltage

  - __________ determines polarity and angle to flux lines also controls amount of voltage.

- Faraday's Law

  $E = N\frac{d0}{dt}$ (volts) $\frac{d0}{dt}$ = differential change in mag flux through coil @ particular time

★ Will a stationary coil sitting in a mag field induce voltage? Why or why not?

Polarity of induced voltage across coil opposes increasing current. This is a choking action described by Lenz's Law.

★ What is Lenz's Law?

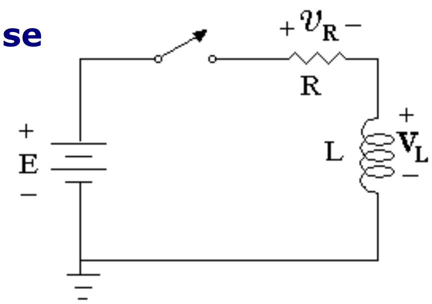
$$L = N\frac{d0}{di_L} \quad \therefore \quad e = N\frac{d0}{dt} = \left(N\frac{d0}{\cancel{di_L}}\right)\left(\frac{\cancel{di_L}}{dt}\right) \rightarrow eL = L\frac{di_L}{dt}$$

$$\boxed{v_L = L\frac{di_L}{dt}}$$

Same eq. but written for inductors

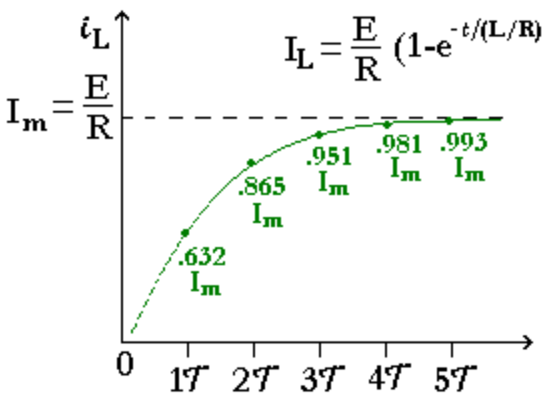
★ If there is no change in current across inductor, what will happen to V_L?

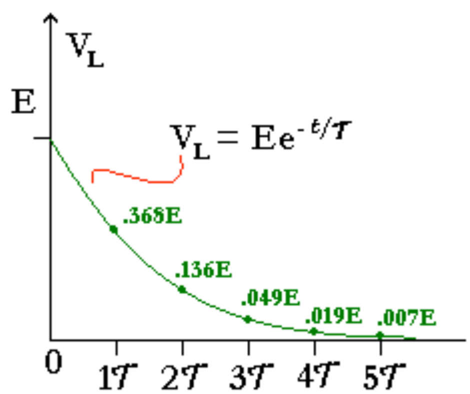
D. R-L Transients: The Storage Phase

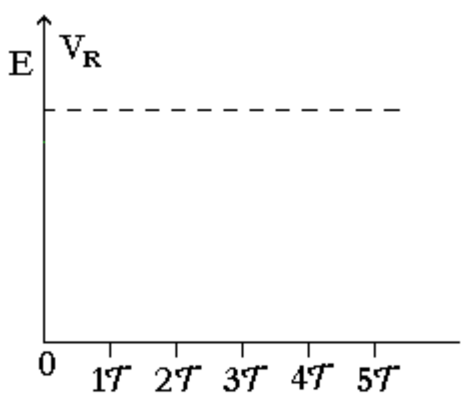


- Energy is stored in form of magnetic field linking the coil.

@ instant of switch closure







★ Draw the voltage curve for the resistor.

$$\tau = \frac{L}{R}$$

$$i_L = \frac{E}{R}(1-e^{-t/\tau}) \text{ (amperes, A)}$$

$$v_L = Ee^{-t/\tau}$$

$$v_R = E(1-e^{-t/\tau})$$

E = voltage
R = resistance
τ = time constant
t = time of interest

★ Why does the shape for the V_R curve match the inductor current curve?

- The ___________ cannot change instantaneously in an inductive network

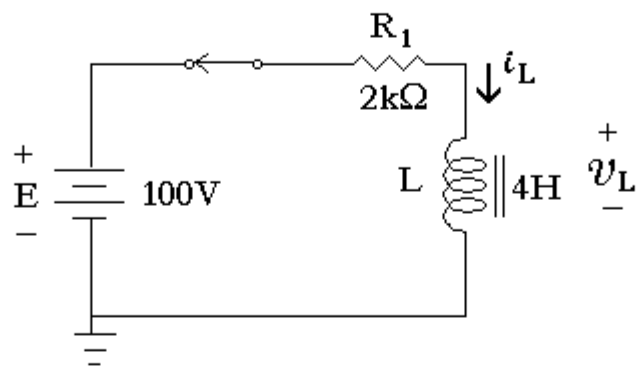
- Inductors act as ___________ ___________ at the instant of switch closure.

- Inductor takes on characteristics of a ___________ _________ When steady-state conditions have been established.

❋ Example:

Find the expression for the following circuit for the transient behavior of I_L and v_L

if the switch is closed at t=0 s. Sketch the resulting curves.



E. Initial Conditions

Current through a coil can't change instantly, so it begins its initial transient value at the initial value established by network prior to switch closure.

★ Draw this.

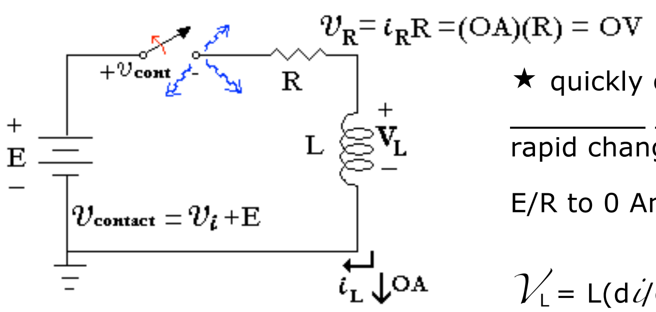
Current eq

$$i_L = I_f + (I_i - I_f)\, e^{-t/\tau}$$

F. R-L Transients: The Release Phase

- Energy stored in form of magnetic field established by the current through the coil

- Must have current passage to store energy



★ quickly opening switch after ________ _________ _________ are reached causes a rapid change in current from maximum of E/R to 0 Amperes. This means di/dt of

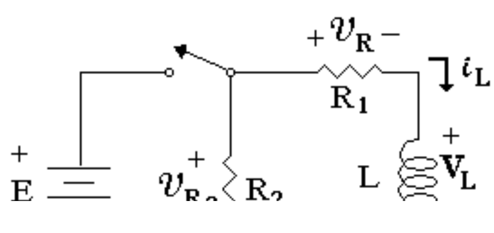
$V_L = L(di/dt)$ is very large for an instant

that makes a large V_L that is added to the

E voltage.

Voltage across reversing polarity conductor (we've Thevenized the circuit)

$$v_L = -(v_{R_1} + v_{R_2})$$



Transient discharge eq. after steady state conditions reached

$v_L = -V_i\, e^{-t/\tau}$

$\tau' = \dfrac{L}{R_T} = \dfrac{L}{R_1 + R_2}$

$i_L = (E/R_1)\, e^{-t/\tau'}$

$v_{R1} = E\, e^{-t/\tau'}$

for Thevenized circuit shown previously

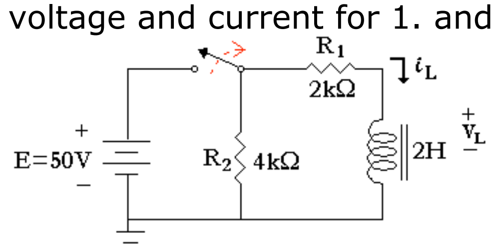
$v_{R2} = -\,(R_2/R_1)\, E\, e^{-t/\tau'}$

❋ Example:

For the following circuit:

1) Find the math expressions for i_L, v_L, v_{R1}, and v_{R2} for 5τ of storage phase
2) Find the math expressions for i_L, v_L, v_{R1}, and v_{R2} if the switch is opened after 5τ of the storage phase
3) Sketch the waveform for each voltage and current for 1. and 2. and define the polarities.

(use back or more paper if necessary)



If steady state conditions are not established during charging and switch is opened:

$$i_L = I_i e^{-t/\tau'}$$

$$v_L = -V_i e^{-t/\tau'} \quad \textbf{w/} \quad v_i = I_i (R_1+R_2)$$

G. Instantaneous Values

$$t = \tau \log_e \frac{(I_i - I_f)}{(i_L - I_f)} \quad \text{(seconds, s)}$$

$$t = \tau \log_e \frac{V_i}{v_L} \quad \text{(seconds, s)}$$

$$t = \tau \log_e \left(\frac{V_f}{V_f - v_R} \right) \quad \text{(seconds, s)}$$

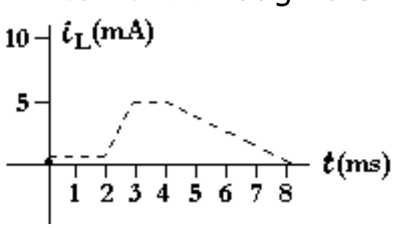
H. Average Induced Voltage

$$v_{L_{av}} = L \frac{\Delta i_L}{\Delta t} \quad \text{(volts, V)}$$

★ What does the □ symbol indicate?

❋ Example:

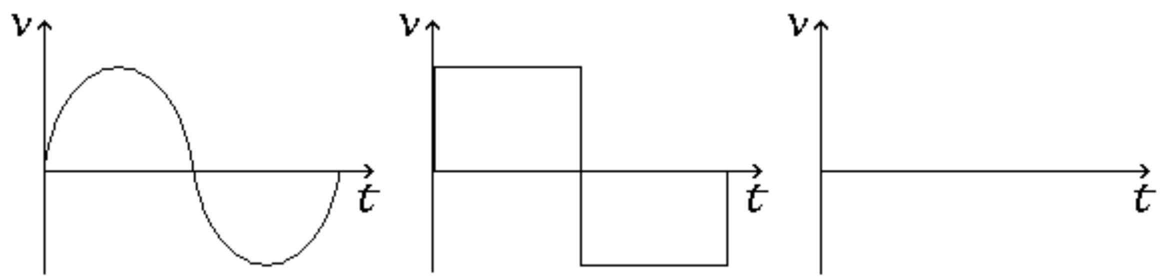
Find the waveform for the average V across the coil. If the current through a 5 mH coil as represented is used.



I. Inductors in Series and Parallel

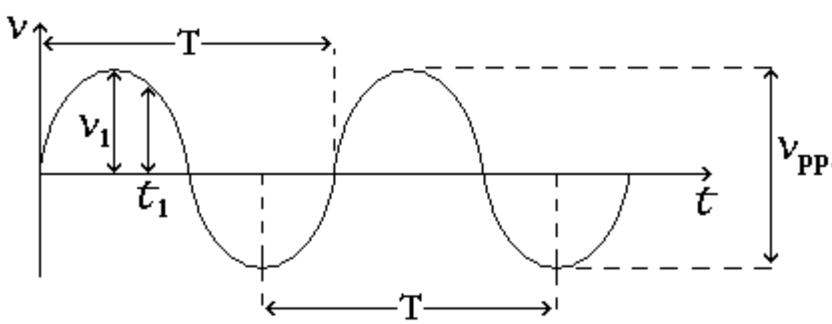
C. Steady State Conditions

Energy Stored by and Inductor Type Alternating Waveforms



Sinusoidal ______________ Triangular

Sinusoidal



Terms

- Waveform
- Instantaneous value
- Peak amplitude
- Peak-to-peak value
- Period
- Cycle

1 hertz (Hz) = (cps)

$f = \frac{1}{T}$

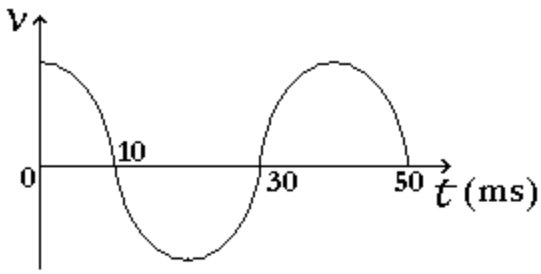
f = Hz
T = seconds (s)

$T = \frac{1}{\quad}$

★ What is the period of a periodic waveform with a frequency of 120 Hz?

T = _____________ =

★ What is the frequency of the waveform below?



f =

★ Low vs. High Frequency Waveforms in a graph

Low vs. High Amplitude Waveforms in a graph

★ Angular Frequency (□)

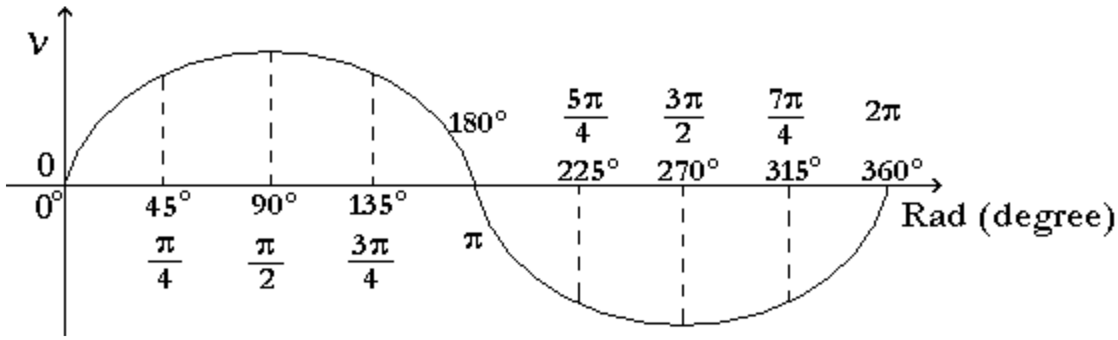
□ = 2□ f = $\frac{2\pi}{T}$ (Radians/second)

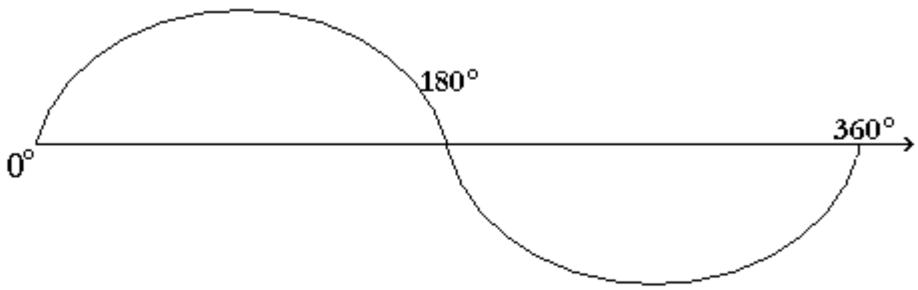
Radii

(number of Radians) = $\left(\frac{\pi}{180^\circ}\right)$ (number of degrees)

★ 90˚ = ($\frac{\pi}{180^\circ}$) 90˚ = □ / 2

★ 30˚ = =





$\omega = \frac{\alpha}{t}$

□ = angular velocity

□ = distance (degrees or radians)

□ =

t = seconds (time)

★ Given f = 60 Hz

Determine how long it will take the sinusoidal waveform to pass through an angle of 45˚

□ = and □ = = Rad = Rad

2□ f = ~> t = ________________

= ________________

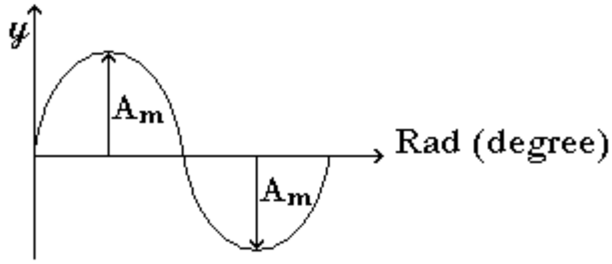
= ________ = _______ = ms

★ Find the angle through which a sinusoidal waveform of 60Hz will pass in a period of 5 ms.

□ = □ t = =

$\square = \qquad = \qquad ^\circ$

Sinusoidal Voltage/ Current



$y = A_m \sin \square$ if we want to express v or i,

$y = A_m \cos \square$ $\therefore\ v = V_m \sin \square\ = V_m \sin \square\, t$

$i = I_m$

Therefore

$\square\ = \sin^{-1} v/V_m$ or $\square\ = \sin^{-1}$ ________

★ Given $v = (30 * 10^{-3} \sin \square)$ V

Determine the angles at which v will be 6 mV

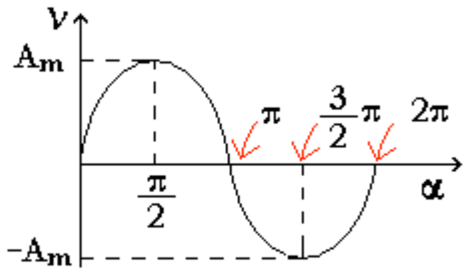
From the expression of v, we find $V_m =$

$$\alpha = \sin^{-1} \frac{v}{V_m}$$

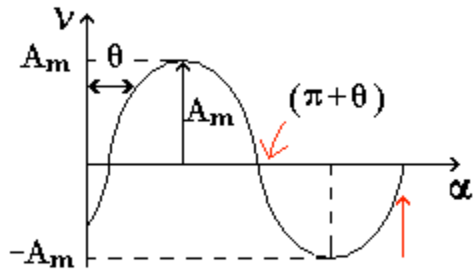
$$\alpha = \sin^{-1} \frac{6\,\mathrm{mV}}{30\,\mathrm{mV}} = \sin^{-1}$$

$\alpha =$ 11.54° and

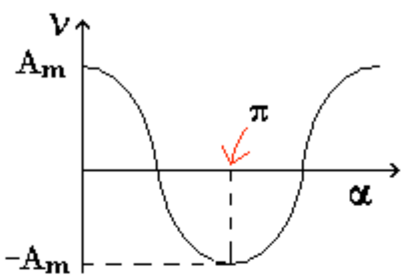
Why do we have two □ s?



$v = A_m \sin \omega t$



$v = A_m \sin(\omega t - \theta)$



$v = A_m \sin(\omega t + 90°)$

$v = A_m \sin(\omega t + \frac{\pi}{2})$

$v = A_m \sin \omega t$

Terms:

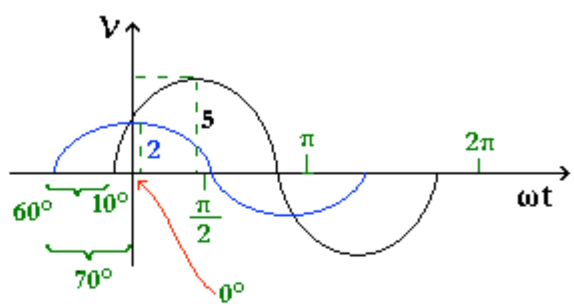
- Leads
- Lags
- In phase

What is the condition?

★ What is the relationship between v and i waveforms below:

a. $v = 5 \sin(\square t + 10°)$

$i = 2 \sin(\square t + 70°)$



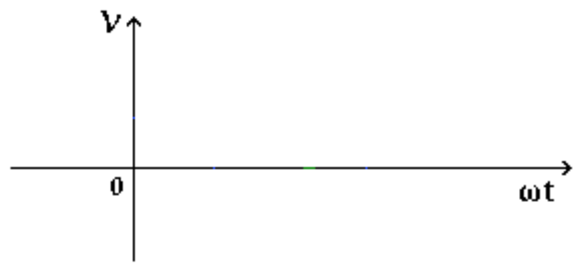
i leads v by 60°

or

v lags i by 60°

b. i = 6 cos (□ t + 10°)

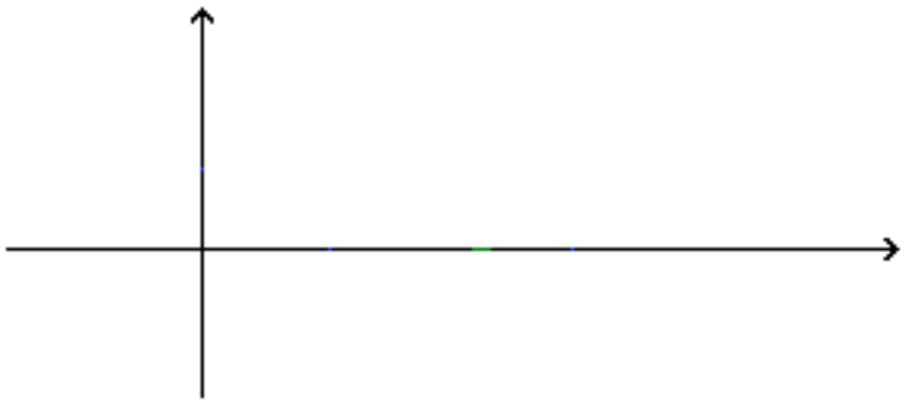
v = 3 sin (□ t - 15°)



______ leads ______ by ______
or
______ lags _______ by ______

c. i = -2 cos (□ t - 60°)

v = 3 sin (□ t - 150°)



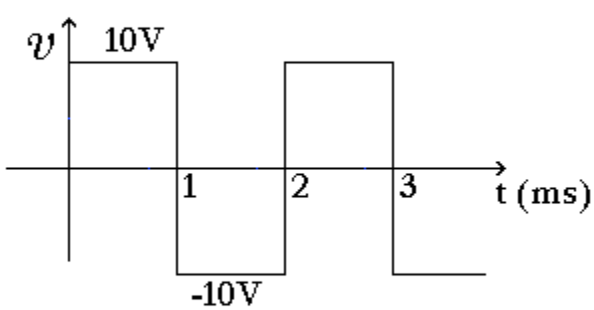
★ What is "Average Value"?

$$G \text{ (average value)} = \frac{\text{Algebraic sum of areas}}{\text{Length of curve}}$$

★ What happens if some area contributions area from below the horizontal axis?

★ Determine the average value of the following waveforms:

a.

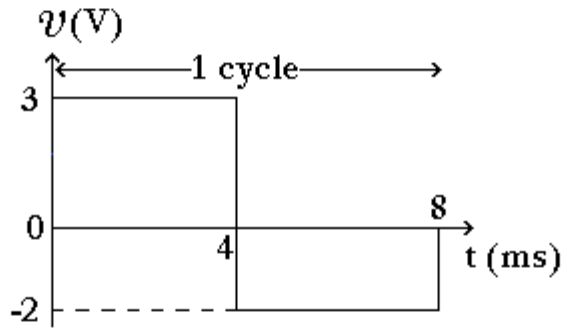


Any condition applied? What?

How to calculate

G = _________________________ = = V

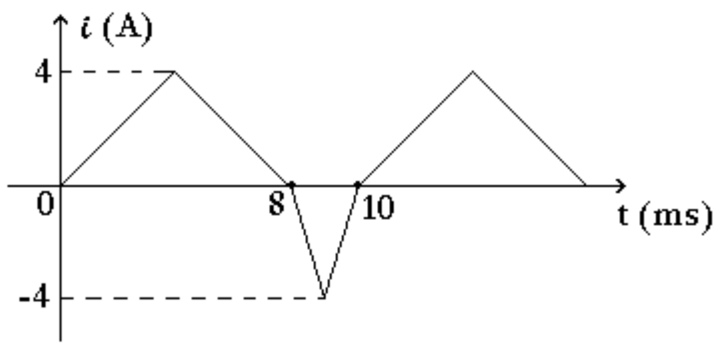
b.



- Condition?
- How to calculate G?

G = ______________________________ = = V

c.

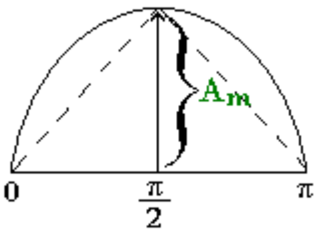


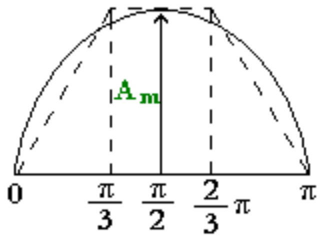
Conditions?

How to calculate G?

G = ______________________ =

For sinusoidal waveforms





Area $= \frac{1}{2}$ b h

$= \dfrac{(\pi)(A_m)}{2}$

$\approx 1.584\ A_m$

Area $= (\square + \square/3)\ (1/2\ A_m)$

$= \dfrac{2}{3}\pi\,(A_m)$

$= 2.094\ A_m$

$$\text{Area} = \int_0^{\pi} A_m \sin\alpha\, d_\alpha$$

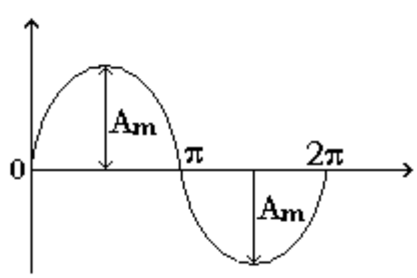
$$= A_m[-\cos\alpha]_0^{\alpha}$$

$$= \qquad\qquad =$$

$\therefore$ $\boxed{\text{Area} = 2\ A_m}$ ~> G = Area/$\square$ = $2A_m/\square$ = 0.637 A_m

Now,

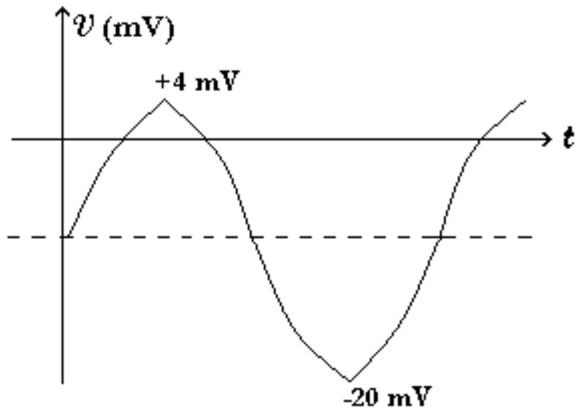
★ Determine the average value of the sinusoidal waveform.



Guess what would be the average time

Why?

What about this waveform?



Peak to peak = mV

Peak amplitude = mV

Average (dc level) = mV

Why?

<u>Effective (RMS) Values</u>

Why RMS value?

$P_{ac} = P_{dc}$

$P_{dc} = I_{dc}^2\ R$

$= \quad (I_m \sin \omega t)^2\ R \quad = \quad (I_m^2 \sin^2 \omega t)\ R$

And

$P_{ac} \quad =$

$\therefore \quad I_{dc}^2 R \quad =$

$$I_{dc} = \frac{I_m}{\sqrt{2}} = 0.707\ I_m$$

$$\sqrt{\frac{\int_0^T i^2\,(t)\,dt}{T}}$$

the area of $i^2(t)$

We can also express I_{RMS} =

What about $\mathcal{V}_{dc}$?

We can conclude: $I_{RMS} = \frac{1}{\sqrt{2}} \mathbf{I_m}$ =

V_{RMS} (E_{RMS}) =

Similarly, $I_m = \sqrt{2}\, I_{RMS} = 1.414\, I_{RMS}$

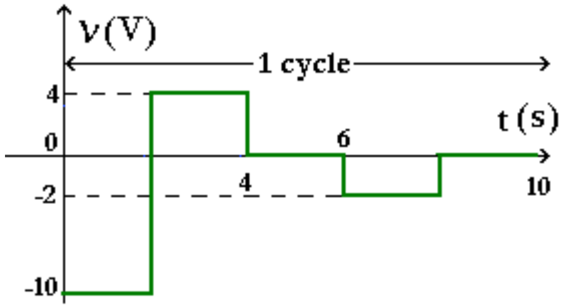
$V_m = \sqrt{2}\, V_{RMS}$ =

★ The 120 V_{dc} source delivers 3.6W to the load.
Determine the peak value of the applied voltage (V_m) and the current (I_m) if the ac source is to deliver the same power to the load.

What is the problem?

How to calculate?

★ Find the RMS value of the waveform below



$$V_{RMS} = \sqrt{\frac{\qquad}{\qquad}}$$

$$= \qquad\qquad\qquad V$$

If you have a waveform that has a dc and an ac component, you should calculate the total RMS value using this formula

$$V_{RMS} = \sqrt{V_{dc}^2 + V_{ac}^2{}_{(RMS)}}$$