# ETE 2200

## Enhanced Guided Notes: Set 12

**Topics:**

A. **Response of Basic R, L, and C Elements to a Sinusoidal Voltage/Current**
B. **Frequency Response**
C. **Average Power and Power Factor**
D. **Rectangular/Polar Form**
E. **Phasors**

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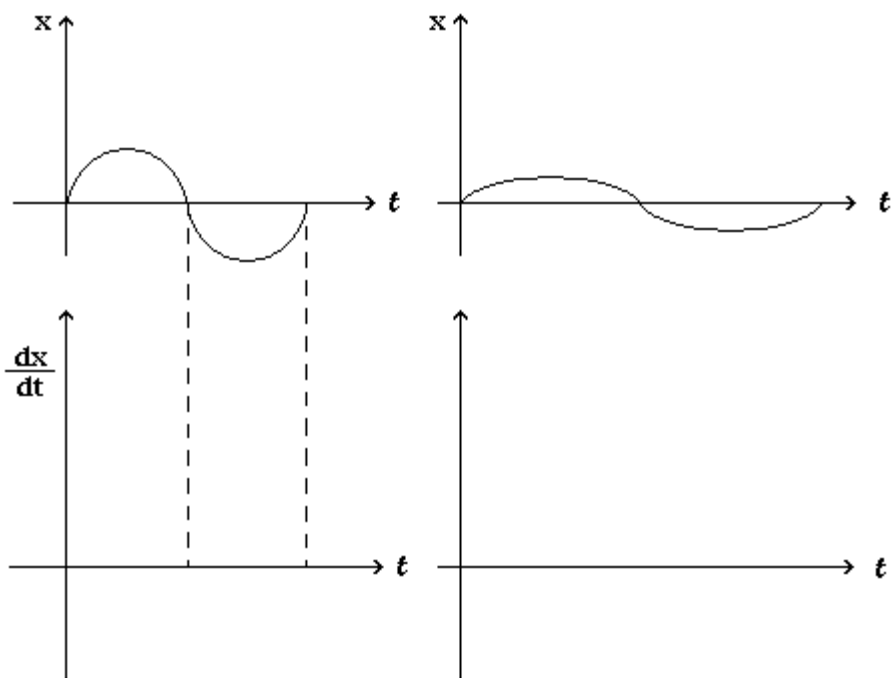
## A. Response of Basic R, L, and C Elements to a Sinusoidal Voltage/Current



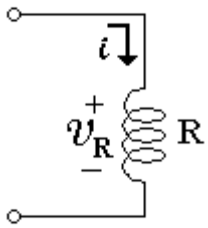

For sinusoidal voltage

$$\mathcal{V}(t) \quad = \quad V_m \sin(\square\, t \pm \square)$$

$$\frac{d\mathcal{V}(t)}{dt} =$$

=

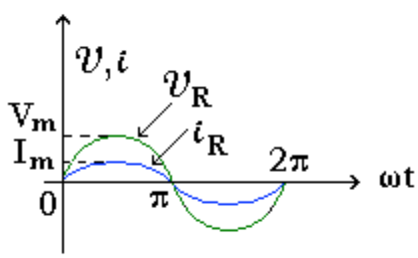
Resistor




For a given $v$ = $V_m$ sin □ t

$\therefore$ $i$ = $v$/R =

Where $I_m$ =




For a given $i$ = $I_m$ sin □ t

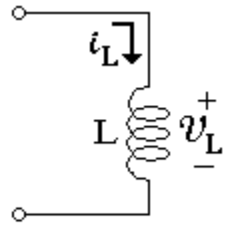
$\therefore$ $v$ = $i$R =

=

Where $V_m$ =

Conclusions:

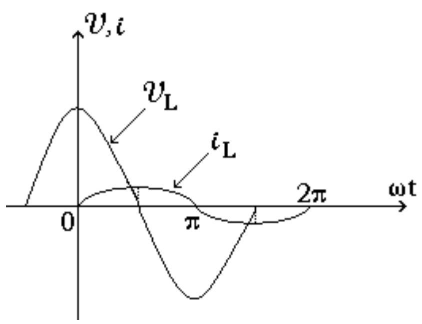
Inductor




Remember $v_L$ = L ________________

If $i_L$ = $I_m$ sin $\omega$ t

Then $v_L$ = L ________ = L d/dt




$v_L$ =

or

$v_L$ =

Now, what if we know $v_L$

Remember iL = _____ ∫ dt

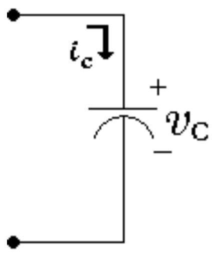
Note, $\omega$ L = Reactance ($X_L$)

∴ $X_L = \omega L$

$X_L = V_m / I_m$

Conclusions:

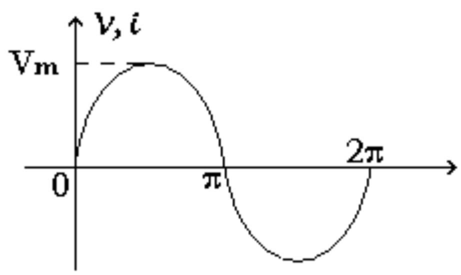
Capacitor




Remember, $i_C$ = C ______________

If $V_C = i_m \sin \omega t$

Then $i_C$ = C d/dt




$i_C$ =

=

If we know $i_C$ ~> $V_C = \quad \int \quad dt$

Note, $1/\omega c$ = reactance ($X_C$)

$\therefore \quad X_C = 1 / \omega c$

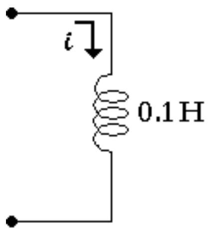
$X_C = V_m / I_m$

Conclusions:

★ The current through a 0.1 H coil is $i$ = 7 sin (377 t - 70$^\circ$)

Find the sinusoidal expression for the voltage across the coil.

Sketch the $v$ and $i$ curves.




How do we approach this problem?

· $v_L$ =

Another way, use $X_L$ =

=

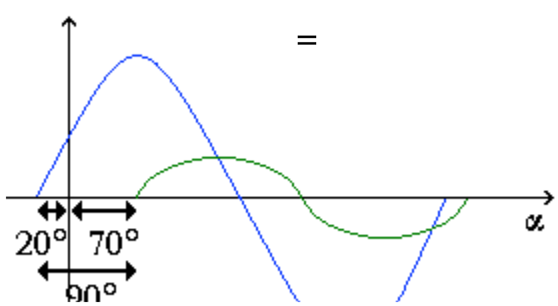
∴ $V_m$ =

=

For inductor, __________ leads ___________ by __________$^\circ$

∴ $v_L$ =

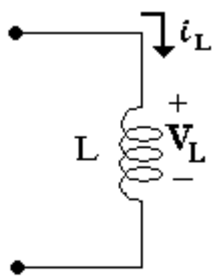
=



★ The current through a 20□ inductive reactance is

$i_L$ = -6 sin (□ t -30°)

What is the sinusoidal expression for the voltage?




$i_L$ = -6 sin (□ t -30°) ~> $i_L$ =

$i_L$ =

What do we need to know to solve this problem?

How do we find it?

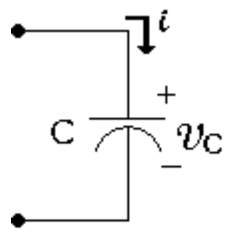
$\therefore \quad v_L \quad =$

Is there another way to solve this problem?
How?

★ The current through a 10□ capacitive reactance is

$$i_C = 2*10^{-6} \sin(□ t + 60^\circ)$$

Write the sinusoidal expression for the voltage.




$i = 2*10^{-6} \sin(□ t + 60^\circ)$

What do we need to know to solve this problem?

How do we find it?

$$\therefore \quad v_L \quad =$$

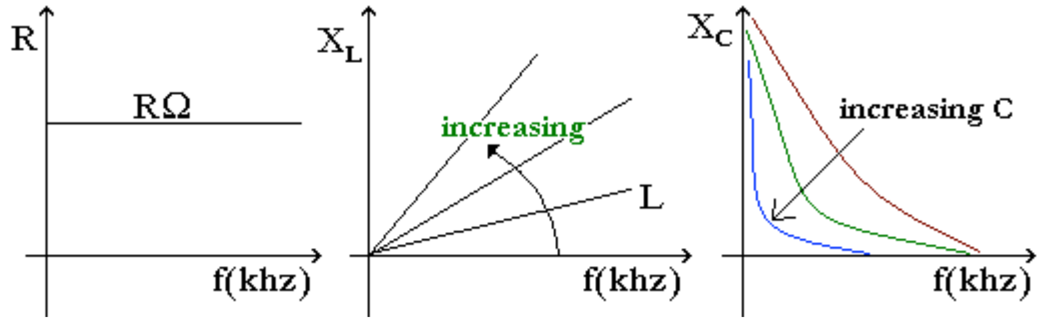
★ For the following pair of voltage and current, indicate whether the element involved is a capacitor, and inductor, or a resistor, and the value of C, L, or R.

$$v = 36 \sin(754t - 80^\circ)$$

$$i = 4 \sin(754t - 170^\circ)$$

How do we solve this problem?

## B. Frequency Response




f = 0 ~> R = R　　f = 0 ~> L □　　f = 0 ~> c □

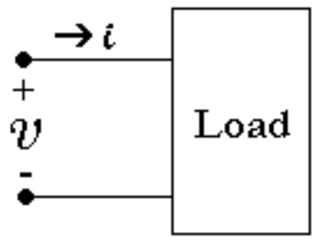
f >> ~> R = R　　f >> ~> L □　　f >> ~> c □

Why?

★ At what frequency will the reactance of a 500 mH inductor match the resistance level of a 5k□ resistor?

## C. Average Power and Power Factor

What will be the net transfer of power/energy? Zero?




<u>Resistive Load</u>

$P_{av} = \dfrac{V_m I_m}{2} = V_{Rms} I_{Rms}$

Where $i$ = I and $V$ = V

<u>Inductive/Conductance Load</u>

$i = I_m \sin(\square\, t + \square\,)$

$V = V_m \sin(\square\, t + \square_r)$

$P = V_m I_m \sin(\square\, t + \square_i)$

We know that $\sin A \sin B = \dfrac{\cos(A-B) - \cos(A+B)}{2}$

$\therefore \quad \sin(\square\, t + \square_i)$

$\therefore \quad P = \dfrac{V_m I_m}{2} \cos \square$

$P = \dfrac{V_m}{\sqrt{2}} \dfrac{I_m}{\sqrt{2}} \cos \square$

$P = V_{eff} I_{eff} \cos \square$
$\quad = V_{Rms} I_{Rms} \cos \square$

Is the magnitude of average power delivered independent of whether $v$ leads $i$ or $i$ leads $v$? Why?

Conclusions:

- In a purely resistive circuit ~> P =
- In a purely inductive circuit ~> P =
- In a purely capacitive circuit ~> P =

★ Find the average power delivered to a network having

$i = 3 \sin(\omega t - 50^\circ)$

$v = 150 \sin(\omega t - 70^\circ)$

How do we solve this problem?

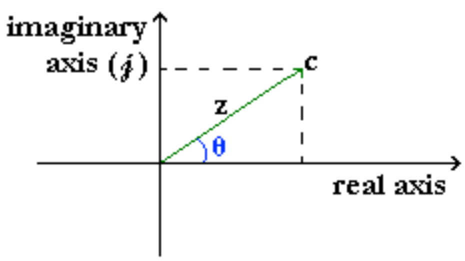
Power Factor (pf) = cos  = ___________________

Two scenarios:

° Resistive Load

° inductive/capacitive load

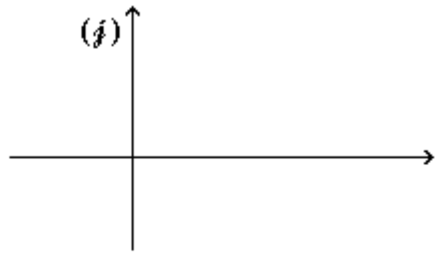
## D. Rectangular / Polar Form



$$C = x \pm jY \quad \Big\} \quad z = \sqrt{x^2 + Y^2}$$

$$C = z \angle \square \qquad \square = \tan^{-1} y/x$$

★ Convert $C = 3 + j4$ to polar form

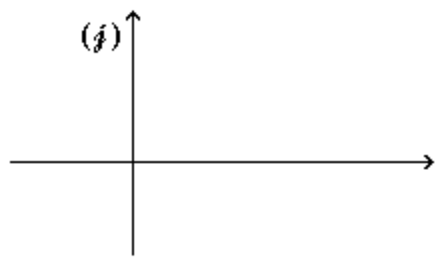



z =

□ = $\tan^{-1}$

∴ C = $\angle$

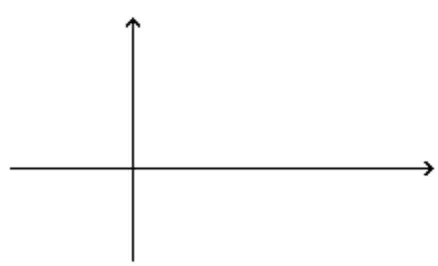
★ What if $C = -3 + j4$




Convert $C = 10 \angle 45^{\circ}$ to rectangular form

X =

Y =



∴ C =

Mathematical Operations with Complex Numbers

For $C_1 = \pm X_1 \pm jY_1$

$C_2 = \pm X_2 \pm jY_2$

Addition $C_1 + C_2 = (\pm X_1 \pm X_2) + j(\pm Y_1 \pm Y_2)$

Subtraction $C_1 - C_2 = (\pm X_1 - (\pm X_2)) + j$

Multiplication $C_1 \cdot C_2 = (X_1X_2 - Y_1Y_2) + j(Y_1X_2 + X_1Y_2)$

Division $C_1/C_2 = \dfrac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j\,\dfrac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2}$

**For $C_1 = Z_1 \angle \theta_1$**
**$C_2 = Z_2 \angle \theta_2$**

Addition $C_1 + C_2 = (Z_1 + Z_2)\angle \theta_1$ or $(Z_1 + Z_2)\angle \theta_2$
Condition?

Subtraction $C_1 - C_2 =$
Condition?

Multiplication $C_1 \cdot C_2 = Z_1 Z_2 \angle \square_1 + \square_2$

Division $C_1 / C_2 = Z_1 / Z_2 \angle \square_1 + \square_2$

## E. Phasors

↓

We use RMS values instead of peak values

Time Domain | Phasor Domain

69.6 sin (□ t + 72°) ~> (0.707)(69.6) ∠72° = 49.21 ∠72°

45 cos □ t ~>

★ Let $\mathcal{V}_a = 50 \sin(377t + 30^\circ)$

$\mathcal{V}_b = 30 \sin(377t + 60^\circ)$

Find $\mathcal{V}_{ab} = \mathcal{V}_a + \mathcal{V}_b$

What do we need to do now?

$\mathcal{V}_a =$ $\rightarrow \mathbb{V}_a =$ $\angle$

$\mathcal{V}_b =$ $\rightarrow \mathbb{V}_b =$ $\angle$

$\mathbb{V}_a =$

$\mathbb{V}_b =$

$\therefore \mathbb{V}_{ab} = \mathbb{V}_a + \mathbb{V}_b =$

$\mathcal{V}_{ab} =$